Résumé: Energy estimation of earthquake faulting processes

Shiro Hirano

In this résumé, we treat a time-dependent elastic equation in a domain containing a surface on which the solution is discontinuous. This hyperbolic equation has widely been employed to model earthquakes occurring in the Earth as well as seismic waves radiated from such earthquakes. We especially focus on a framework using boundary data for estimating kinetic, potential, and dissipated energies.

1 Introduction

On a vector field $\boldsymbol{u} : \Omega \times \mathbb{R}^1 \ni (\boldsymbol{x}, t) \mapsto \boldsymbol{u}(\boldsymbol{x}, t) \in \mathbb{R}^3$ and its time-derivative $\boldsymbol{v} := \partial_t \boldsymbol{u}$, a governing equation of a linear elastic body $\Omega(\subset \mathbb{R}^3)$ is defined as the following partial differential equation (PDE):

$$\begin{cases} \rho\, \partial_t \boldsymbol{v} = \nabla \cdot \sigma, \\ \partial_t \sigma = \boldsymbol{C} \partial_t \varepsilon, \\ \partial_t \varepsilon = \frac{1}{2}\left(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T\right) \end{cases} \Leftrightarrow \begin{cases} \rho\, \partial_t v_i = \sum_{j=1}^3 \partial_{x_j} \sigma_{ij}, \\ \partial_t \sigma_{ij} = \frac{1}{2} \sum_{k,l=1}^3 c_{ijkl}\, \partial_t \varepsilon_{kl}, \\ \partial_t \varepsilon_{ij} = \frac{1}{2}\left(\partial_{x_j} v_i + \partial_{x_i} v_j\right), \end{cases} \tag{1}$$

where $i, j, k, l = 1, 2, 3$. Phenomenologically, $\boldsymbol{u}$, ε, and σ are regarded as displacement, strain, and stress, respectively, at the interior points or the surface of Ω; $\rho = \rho(\boldsymbol{x}) > 0$ is the density of a material; and $\boldsymbol{C} = (c_{ijkl}(\boldsymbol{x}))$ is a fourth-order tensor called elasticity tensor. In general, three symmetries of the elasticity tensor, that is, $c_{ijkl} = c_{jikl}$, c_{ijlk}, and $= c_{klij}$, are derived owing to the conservation of angular momentum, definition of strain tensor, and existence of potential energy function, respectively (e.g., [2]). If $\boldsymbol{C}$ is isotropic, it can be reduced to

$$c_{ijkl} = \lambda\, \delta_{ij}\, \delta_{kl} + 2\, \mu\, \left(\delta_{il} + \delta_{jk}\right)$$

where positive quantities λ and μ are called Lamé's constants, and δ_{ij} is Kronecker's delta. Furthermore, if the quantities are uniform constants, a hyperbolic PDE called Navier's equation is obtained:

$$\partial_t^2 \boldsymbol{u} = \frac{\lambda + 2\mu}{\rho} \nabla(\nabla \cdot \boldsymbol{u}) - \frac{\mu}{\rho} \nabla \times (\nabla \times \boldsymbol{u}).$$

College of Science and Engineering, Ritsumeikan University, 1-1-1, Noji-Higashi, Kusatsu-city, Shiga, 525-8577, Japan. See https://interfacial.jp/ for contact details.

Unlike a scalar wave equation or Maxwell's equation, Navier's equation consists of two waves: dilatational and shear waves with propagation velocities of $\sqrt{(\lambda+2\mu)/\rho}$ and $\sqrt{\mu/\rho}$, respectively. Various physics accompanied by wave propagation within solids can be modelled using the equation. The modelling of seismic waves caused by earthquakes and propagating through the Earth is a controversial problem with some difficulties including spatial heterogeneity of elastic tensor $\boldsymbol{C}(\boldsymbol{x})$, and the observability of data including displacement $\boldsymbol{u}$ and velocity $\boldsymbol{v}$ only at the Earth's surface $\partial\Omega$. Moreover, earthquakes are considered as the discontinuity of material suddenly occurring in the Earth's interior; in other words, they are transient phenomena where discontinuity of $\boldsymbol{u}$ occurs, grows, and stops within Ω.

In this résumé, we introduce the modelling of this phenomena in seismology, and mention a mathematical framework to estimate the energy change in a system based on observed data $\boldsymbol{u}$ at $\partial\Omega$. In this regard, we introduce a complete form of PDE; an energy conservation law; a relation among kinetic, potential, and dissipated energies, and a transient source process. After adding some views of energetics to the contents from [5], we briefly summarize the results of [6].

2 Formulation of PDE

We define a PDE satisfied in a bounded domain $\Omega(\subset \mathbb{R}^3)$ with a sufficiently smooth boundary. Here, a finite smooth surface Γ is embedded in Ω, and the definition of vector field $\boldsymbol{u}$ is slightly modified as follows: $\Omega\backslash\Gamma\times\mathbb{R}^1 \ni (\boldsymbol{x},t) \mapsto \boldsymbol{u}(\boldsymbol{x},t) \in \mathbb{R}^3$. Then, $\boldsymbol{D}$, which is a distribution of discontinuity of $\boldsymbol{u}$ at $\boldsymbol{\xi} \in \Gamma$, is defined as

$$\boldsymbol{D}(\boldsymbol{\xi},t) := \lim_{\epsilon\to 0}\left[\boldsymbol{u}(\boldsymbol{x},t)\right]_{\boldsymbol{x}=\boldsymbol{\xi}-\epsilon\boldsymbol{\nu}^\Gamma}^{\boldsymbol{x}=\boldsymbol{\xi}+\epsilon\boldsymbol{\nu}^\Gamma} = \left[\boldsymbol{u}(\boldsymbol{x},t)\right]_{\boldsymbol{x}\to\boldsymbol{\xi}\in\Gamma^-}^{\boldsymbol{x}\to\boldsymbol{\xi}\in\Gamma^+},$$

where Γ^+ and Γ^- are the front and back sides of Γ, respectively, and $\boldsymbol{\nu}^\Gamma$ $(\in\mathbb{R}^3)$ is a unit normal to Γ. In addition, $\boldsymbol{D}$ can be defined uniquely if the Dirichlet boundary value on Γ is given; however, the vice versa is not possible. Hence, given $\boldsymbol{D}$ is insufficient to provide a solution of a boundary value problem and by assuming the law of action and reaction, we obtain

$$\left[\sigma(\boldsymbol{x},t)\boldsymbol{\nu}^\Gamma\right]_{\boldsymbol{x}\to\boldsymbol{\xi}\in\Gamma^-}^{\boldsymbol{x}\to\boldsymbol{\xi}\in\Gamma^+} = 0$$

following a physical requirement. If the system was in static equilibrium before the non-zero value of $\boldsymbol{D}$ emerged, $\boldsymbol{v}$ is considered as the solution for the following initial-boundary value problem with a given $\boldsymbol{D}$:

$$\begin{cases} \rho\,\partial_t\boldsymbol{v} = \nabla\cdot\sigma, & \boldsymbol{x}\in\Omega\backslash\Gamma, t>0 \\ \partial_t\sigma = \boldsymbol{C}\partial_t\varepsilon, & \boldsymbol{x}\in\Omega\backslash\Gamma, t>0 \\ \partial_t\varepsilon = \frac{1}{2}\left(\nabla\boldsymbol{v}+(\nabla\boldsymbol{v})^T\right), & \boldsymbol{x}\in\Omega\backslash\Gamma, t>0 \\ \left[\boldsymbol{u}(\boldsymbol{x},t)\right]_{\boldsymbol{x}\to\boldsymbol{\xi}\in\Gamma^-}^{\boldsymbol{x}\to\boldsymbol{\xi}\in\Gamma^+} = \boldsymbol{D}(\boldsymbol{\xi},t), & \boldsymbol{\xi}\in\Gamma, t>0 \\ \left[\sigma(\boldsymbol{x},t)\boldsymbol{\nu}^\Gamma\right]_{\boldsymbol{x}\to\boldsymbol{\xi}\in\Gamma^-}^{\boldsymbol{x}\to\boldsymbol{\xi}\in\Gamma^+} = 0, & \boldsymbol{\xi}\in\Gamma, t>0 \\ \lim_{\boldsymbol{x}\to\boldsymbol{s}}\sigma(\boldsymbol{x})\,\boldsymbol{\nu}^{\partial\Omega} = 0, & \boldsymbol{s}\in\partial\Omega, t>0 \\ \boldsymbol{v}=0,\ \sigma=\sigma_0,\ \varepsilon=0, & t=0 \end{cases} \tag{2}$$

where $\boldsymbol{\nu}^{\partial\Omega}$ is a unit normal to $\partial\Omega$, and σ_0 satisfies $\nabla\cdot\sigma_0 = 0$ for $t=0$.

From a physical point of view, Γ is a crack, called fault surface, embedded in the Earth, and $\boldsymbol{D}$ is the discontinuity of displacement along the fault. In general, $\boldsymbol{D}\cdot\boldsymbol{\nu}^{\Gamma}$ represents the opening of a crack if it is positive. However, in our case, we assumed $\boldsymbol{D}\cdot\boldsymbol{\nu}^{\Gamma}=0$ because faults cannot open under a highly compressed undergrounded state, and only a slip along the fault can be allowed. In this study, only the $\boldsymbol{D}$ related to the Dirichlet boundary value is given explicitly, and the Neumann boundary value $\sigma\boldsymbol{\nu}^{\Gamma}$ is defined uniquely as well as implicitly by using a linear Dirichlet-to-Neumann operator. In addition, the Neumann boundary value on Γ can be given explicitly in other mechanics problems considering the traction on the fault surface, as reviewed by Hirano [5].

3 Energy conservation law

Here, we take the inner product of the first equation in (2) and $\boldsymbol{v}$ over $\Omega\backslash\Gamma$. First, from the left-hand side, we obtain

$$\int_{\Omega\backslash\Gamma}\rho\,(\partial_t\boldsymbol{v})\cdot\boldsymbol{v}\,d\boldsymbol{x}=\int_{\Omega\backslash\Gamma}\frac{\rho}{2}\partial_t\,|\boldsymbol{v}|^2\,d\boldsymbol{x}.$$

Then, from the right-hand side, we obtain

$$\begin{aligned}\int_{\Omega\backslash\Gamma}(\nabla\cdot\sigma)\cdot\boldsymbol{v}\,d\boldsymbol{x}&=\int_{\Omega\backslash\Gamma}\nabla\cdot(\sigma\boldsymbol{v})\,d\boldsymbol{x}-\int_{\Omega\backslash\Gamma}\mathrm{tr}\,(\sigma\partial_t\varepsilon)\,d\boldsymbol{x}\\&=\int_{\Gamma}\partial_t\boldsymbol{D}\cdot\left(\sigma\boldsymbol{\nu}^{\Gamma}\right)\,d\boldsymbol{\xi}-\int_{\Omega\backslash\Gamma}\frac{1}{2}\partial_t\,\mathrm{tr}\,(\sigma\varepsilon)\,d\boldsymbol{x},\end{aligned}$$

where we apply the divergence theorem to the first term as follows:

$$\begin{aligned}\int_{\Omega\backslash\Gamma}\nabla\cdot(\sigma\boldsymbol{v})\,d\boldsymbol{x}&=\int_{\partial\Omega}\boldsymbol{v}\cdot\left(\sigma\boldsymbol{\nu}^{\partial\Omega}\right)\,d\boldsymbol{\xi}+\int_{\Gamma^+}\boldsymbol{v}\cdot\left(\sigma\boldsymbol{\nu}^{\Gamma}\right)\,d\boldsymbol{\xi}+\int_{\Gamma^-}\boldsymbol{v}\cdot\left(\sigma\left(-\boldsymbol{\nu}^{\Gamma}\right)\right)\,d\boldsymbol{\xi}\\&=\int_{\Gamma}[\boldsymbol{v}]_{\xi\in\Gamma^-}^{\xi\in\Gamma^+}\cdot\left(\sigma\boldsymbol{\nu}^{\Gamma}\right)\,d\boldsymbol{\xi}\\&=\int_{\Gamma}\partial_t\boldsymbol{D}\cdot\left(\sigma\boldsymbol{\nu}^{\Gamma}\right)\,d\boldsymbol{\xi}\end{aligned}$$

with $\sigma\boldsymbol{\nu}^{\partial\Omega}=0$. Further, to the second term, we apply

$$\mathrm{tr}\,(\sigma\,\partial_t\varepsilon)=\sum_{i,j,k,l}c_{ijkl}\,\varepsilon_{kl}\,\partial_t\varepsilon_{ij}=\frac{1}{2}\partial_t\sum_{i,j,k,l}c_{ijkl}\,\varepsilon_{kl}\,\varepsilon_{ij}=\frac{1}{2}\partial_t\,\mathrm{tr}\,(\sigma\varepsilon)\,.$$

Thus, the energy conservation law is obtained as follows:

$$\int_{\Gamma}\partial_t\boldsymbol{D}\cdot\left(\sigma\boldsymbol{\nu}^{\Gamma}\right)\,d\boldsymbol{\xi}-\int_{\Omega\backslash\Gamma}\frac{1}{2}\partial_t\,\mathrm{tr}\,(\sigma\varepsilon)\,d\boldsymbol{x}-\int_{\Omega\backslash\Gamma}\frac{\rho}{2}\partial_t\,|\boldsymbol{v}|^2\,d\boldsymbol{x}=0,\qquad(3)$$

where the first term on the LHS presents energy flux through Γ; the second term is an analog of $\int\frac{1}{2}\partial_t\,|\nabla\boldsymbol{u}|^2\,d\boldsymbol{x}$ and represents the rate of potential energy change; and the third term represents the rate of kinetic energy change within the domain. In terms of physics, fracture and friction between rock masses may occur along the fault Γ; thus, the energy flux can be interpreted as a process of energy dissipation due to fracture and friction. Hence, eq. (3) implies that the potential energy released from the domain (second term) is partially dissipated on the fault surface (first term), and the residue can be converted into the kinetic energy (third term).

4 Basic concepts of energy estimation based on order estimation and asymptotic analysis

If we can estimate two terms on the LHS of eq. (3), the other can be determined consequently. However, the physical estimation of the terms is not straightforward because $\boldsymbol{v}$ can be observed only at $\boldsymbol{x} \in \partial\Omega$ (i.e., the Earth's surface far away from the fault Γ). In the following, we consider the asymptotics of the integrands in eq. (3) and approximate the integrals by using the data at $\partial\Omega$.

In addition, we consider an order of a solution for a hyperbolic PDE with the radiation condition as $|\boldsymbol{u}| = O(r^{-1})$ for distance r sufficiently farther from the source. Based on $d\boldsymbol{x} = r\,dr\,d\Omega$ ($d\Omega$: the solid angle), the densities of potential and kinetic energies obtain the asymptotic forms

$$\mathrm{tr}\,(\sigma\,\varepsilon)\,r = \mathrm{tr}\,(\nabla u)^2\,r = O\left(r^{-3}\right),$$

and

$$|\boldsymbol{v}|^2\;r = O\left(r^{-1}\right),$$

respectively. These imply that the kinetic energy can be propagated far field, while the potential energy is dominantly stored within the near field. As $\boldsymbol{u}$ and $\boldsymbol{v}$ are observable only at far-field, the estimation of the kinetic energy should be relatively achievable.

First, we derived an approximation of the kinetic energy. As shown in the Introduction, the governing equation of elasticity is hyperbolic; thus, the consideration of a fundamental solution G of a scalar wave equation

$$\Box G = 4\pi M(t)\partial_{\boldsymbol{\nu}^\Gamma}\delta(\boldsymbol{x}) \tag{4}$$

with the radiated condition instead of eq. (1) may facilitate in the order estimation and approximation. In eq. (4), M is a function whose derivative $\partial_t M$ is non-negative and compactly supported, and δ is Dirac's delta function. In addition, Γ is assumed to be localized only at the origin, and $\partial_{\boldsymbol{\nu}^\Gamma}$ represents a normal derivative on Γ. Burridge & Knopoff [4] and Aki & Richards [2] showed that earthquake faulting processes, which are transient and irreversible, can be equivalent to the inhomogeneous term in eq. (4) even in an elastic case. A far-field asymptotic solution of eq. (4) is given as

$$G(\boldsymbol{x},t) = \partial_{\boldsymbol{\nu}^\Gamma}\frac{M(t-r/c)}{r} = O\left(\partial_t M r^{-1}\right) \tag{5}$$

where r is a large distance and c is the wave propagation velocity. Further details on the estimation of far-field kinetic energy can be found in [3], [7], and [9]. Essentially, the methods in these studies are based on an integration of kinetic energy density over sphere S, which encloses and is far away from Γ. The total kinetic (or radiated) energy E_R can be represented as [3, 7, 9]

$$E_R = \int_0^\infty \left(\int_S |\boldsymbol{v}|^2 \, dS \right) dt = \int_0^\infty \left| \partial_t^2 M \right|^2 \, dt \, \frac{4\pi r^2}{r^2}.$$

Note that G is an analog of $\boldsymbol{u}$; thus, $\boldsymbol{v} = \partial_t \boldsymbol{u}$ is approximated using $\partial_t G = \partial_t^2 M / r$. Actually, the integrand is compactly supported owing to the transient source; therefore, the time-integration is sufficient with a finite interval. The Fourier transform with respect to t, defined as

$$\mathcal{F}_t M(\omega) := \int M(t) e^{-i\omega t} \, dt$$

and Parseval's theorem produce another representation of E_R as follows:

$$E_R = \int_{\mathbb{R}} \omega^2 \left| \mathcal{F}_t \left(\partial_t M \right) (\omega) \right|^2 \, d\omega. \tag{6}$$

Thus, the integration form of the kinetic energy observable at far-field sites is obtained based on the transient source property $\partial_t M$.

Next, we mention a relationship between the far-field observation and energy dissipation on Γ represented as the first term of eq. (3). As seen in the previous paragraph, the potential energy term is barely estimated at far-field sites directly. Hence, by estimating the energy dissipation, the potential energy term can be estimated using the conservation law. The integrand of the dissipation term consists of $\partial_t \boldsymbol{D}$ and $\sigma \boldsymbol{\nu}^\Gamma$ defined on Γ. Aki & Richards [2] revealed that $\partial_t \boldsymbol{D}$ is proportional to $\partial_t M$ defined in the previous paragraph, which can be observed as the far-field solution using eq. (5). Therefore, $\partial_t \boldsymbol{D}$ is now observable at far-field sites. After obtaining the value of $\partial_t \boldsymbol{D}$, the Neumann boundary value $\sigma \boldsymbol{\nu}^\Gamma$ can be calculated using a linear Dirichlet-to-Neumann operator. More details on this procedure can be found in [5].

Although further discussions on the rate of energy dissipation and potential energy change appearing in the first and second terms of eq. (3) are our future work, we can consider the rate-independent energy dissipation E_S on Γ defined as

$$E_S = \frac{1}{2} \lim_{t \to \infty} \int_\Gamma \boldsymbol{D} \cdot \sigma \boldsymbol{\nu}^\Gamma \, d\boldsymbol{\xi},$$

and the total potential energy change ΔW defined as

$$\Delta W := \frac{1}{2} \lim_{t \to \infty} \int_{\Omega \setminus \Gamma} \operatorname{tr} (\sigma \varepsilon) \, d\boldsymbol{\xi}.$$

In the following, we assume that the fault surface is free of traction after an earthquake (i.e., $\lim_{t \to \infty} \sigma \boldsymbol{\nu}^{\boldsymbol{\Gamma}} = -\sigma_0 \boldsymbol{\nu}^{\boldsymbol{\Gamma}}$ on Γ), for simplicity; see [6] for a general case. As with the derivation of eq. (3), by taking the inner product of the governing equation and $\boldsymbol{u}$ instead of $\boldsymbol{v}$, the following holds:

$$\rho \int_{\Omega \setminus \Gamma} \partial_t \boldsymbol{v} \cdot \boldsymbol{u} \, d\boldsymbol{x} - \int_\Gamma \boldsymbol{D} \cdot \sigma \boldsymbol{\nu}^\Gamma \, d\boldsymbol{\xi} - \int_{\Omega \setminus \Gamma} \operatorname{tr} (\sigma \varepsilon) \, d\boldsymbol{x} = 0. \tag{7}$$

As the source process is transient, the integrand of the first term is compactly supported, and the integration is with respect to the volume of the wave-front, which gives a constant evaluated as

$$\int_{\Omega\backslash\Gamma} \partial_t \boldsymbol{v} \cdot \boldsymbol{u}\, d\boldsymbol{x} = \int_{\text{wavefront}} O\left(\frac{1}{r^2}\right) dS = \frac{4\pi r^2}{r^2},$$

for a sufficiently large value of r. Therefore, eq. (7) is equivalent to the following relation:

$$-E_S = \Delta W - const.$$

As described earlier, $\boldsymbol{D}$ can be estimated through far-field observations, while the estimation of the potential change is difficult. Hence, E_S is worth considering for the energetics of earthquake processes, and many researchers have discussed it. By assuming that the direction of $\boldsymbol{D}$ is uniform on a flat fault $\Gamma(\subset \mathbb{R}^2)$, Andrews [1] derived that the following holds:

$$E_S = \frac{1}{2}\int_0^{\infty}\left(\int_{-\pi}^{+\pi} k^2 \left|\mathcal{F}_{\xi}\boldsymbol{D}(\boldsymbol{k})\right|^2 d\theta\right) dk, \tag{8}$$

where the 2-D Fourier transform

$$\mathcal{F}_{\xi}\boldsymbol{D}(\boldsymbol{k}) := \int_{\mathbb{R}^2} \boldsymbol{D}(\boldsymbol{x}) e^{-2\pi i \boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{x}$$

and Parseval's theorem are applied, and $\boldsymbol{k} = (k\cos\theta, k\sin\theta)^T$ (i.e., $k := |\boldsymbol{k}|$).

5 Difficulty in the actual estimation of E_R and E_S

Empirically, the power spectrum densities of $\partial_t M$ and $\boldsymbol{D}$ appearing in the integrands of eq. (6) and (8), respectively, are known to be approximated as follows:

$$\left|\mathcal{F}_t \partial_t M(\omega)\right|^2 = \frac{1}{\{1+\omega^2\}^{1+H}},$$

$$\int_{-\pi}^{+\pi} \left|\mathcal{F}_{\xi}\boldsymbol{D}(\boldsymbol{k})\right|^2 d\theta = \frac{1}{\{1+k^2\}^{1+H}},$$

where H is called the Hurst exponent, and some coefficients are neglected. Accordingly, Hirano & Yagi [6] pointed out that both E_R and E_S have the similar integral representation with the form of

$$I = \int_0^X \frac{x^2 dx}{\{1+x^2\}^{1+H}} = \frac{1}{2}B\left(\frac{1}{1+X^{-2}}; \frac{3}{2}, H - \frac{1}{2}\right)$$

where B is the incomplete beta function, and the upper bound of the integration, X, is introduced because actual data lacks shorter wavelength (i.e., larger x) components owing to dissipation and scattering in the Earth. This result reveals a serious problem on energy estimation with seismic data in [6]. For $H = 1$, we can estimate approximately 90% of the total energy if $X = 10$, which is a usual upper bound. However, for

$H = 0.6$, we estimate at most 30% of the total energy with the same X. Both $H = 1$ and $= 0.6$ have been reported for actual earthquakes and seismic waves; thus, the total energies related to earthquake faulting processes might be severely underestimated in some cases.

Another interesting topic involves fractal dimension of $\boldsymbol{D}(\boldsymbol{\xi})$. The Hurst exponent H is related to fractal dimension d of the distribution (e.g., [8]) as

$$d = 3 - H.$$

According to the definition of H, the Hurst exponent reflects relative power of shorter wavelength components. In other words, the larger the fractal dimension d, the more heterogeneous is the slip distribution $\boldsymbol{D}$. Thus, energies related to earthquakes may reflect heterogeneity and fractalness of environments and mechanisms in the Earth.

Acknowledgment

I appreciate some insightful comments by a reviewer. This is a post-peer-review, pre-copyedit version of an article to be published in *Singularity and asymptotic behavior of solutions for partial differential equations with conservation law: RIMS Kôkyûroku Bessatsu.*

References

[1] Andrews, D. J., A stochastic fault model 1. static case. *J. Geophys. Res.*, **85** (1980), 3867–3877.

[2] Aki, K. & Richards, P. G., *Quantitative seismology*. 1st ed., Freeman, 1980.

[3] Boatwright, J., A spectral theory for circular seismic sources; simple estimates of source dimension, dynamic stress drop and radiated seismic energy. *Bull. Seism. Soc. Am.*, **66** (1980), 1271–1302.

[4] Burridge, R., & Knopoff, L., Body force equivalents for seismic dislocations. *Bull. Seism. Soc. Am.*, **54** (1964), 1875–1888.

[5] Hirano, S., Integral representation and its applications in earthquake mechanics: a review. In: van Meurs, P., Kimura, M., Notsu, H. (eds) *Mathematical Analysis of Continuum Mechanics and Industrial Applications II. CoMFoS 2016*. Mathematics for Industry, **30**. Springer, Singapore, 2018.

[6] Hirano, S. & Yagi, Y., Dependence of seismic and radiated energy on shorter wavelength components. *Geophys. J. Int.*, **209** (2017), 1585–1592.

[7] Kanamori, H., Mori, J., Hauksson, E., Heaton, T. H., Hutton, L. K., & Jones, L. M., Determination of earthquake energy release and M_L using *TERRAscope*. *Bull. Seism. Soc. Am.*, **83** (1993), 330–346.

[8] Mai, P. M., & Beroza, G. C., A spatial random field model to characterize complexity in earthquake slip. *J. Geophys. Res.*, **107** (2002), 2308.

[9] Venkataraman, A., & Kanamori, H., Observational constraints on the fracture energy of subduction zone earthquakes. *J. Geophys. Res.*, **109** (2004), B05302.