S51A-2657 : Propagation Velocity of Pulse-Like Rupture with a Time-Weakening Friction Law

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1. Introduction.

1.1. Motivation

Propagation velocity of rupture along a fault is important for...

- Physical source modeling
- Source inversion technique
- Strong ground motion

Description of rupture velocity

Usually, rupture velocity v_r is several ten per cent of the shear wave speed or Rayleigh wave speed...**Why?**

A simple model is better, e.g., linear elasticity, a 2-D flat fault, homogeneous medium, and uniform rupture velocity...

We here consider pulse-like rupture(Fig. 1), which has been observed seismologically [Heaton 1990].

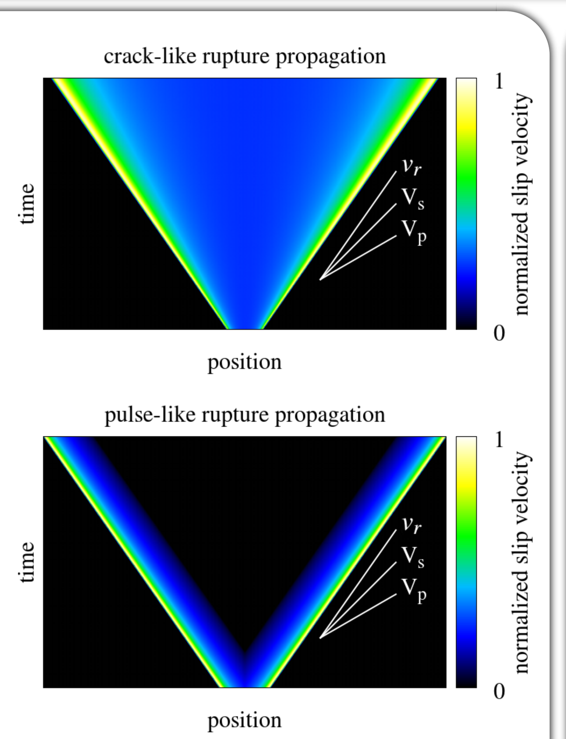

Fig. 1. Schematic illustration of slip velocity distributions corresponding to crack-like and Pulse-like rupture.

How is the propagation velocity of pulse-like rupture determined?

1.2. Outline

Steady-state slip pulse [theoretical, Rice et al. 2005]

+ **Granular layer as slipping plane** [numerical, Hatano 2009]

= **Rupture velocity** as a function of geological parameters

2. Method

2.1. Steady-state dynamic slip pulse model with a slip-weakening friction; Rice et al. [2005] revisited

Assumptions on the fault plane (Fig. 2):

- A pulse, currently slipping region, is moving towards positive direction of x-axis, and its length and propagation velocity are temporally uniform.
- Finite yield stress is archived at the leading edge of the pulse as the maximum static friction.
- Constant shear stress level is archived around the trailing edge of the pulse as the dynamic friction.
- There exists a process zone for relaxation of friction behind the leading edge.

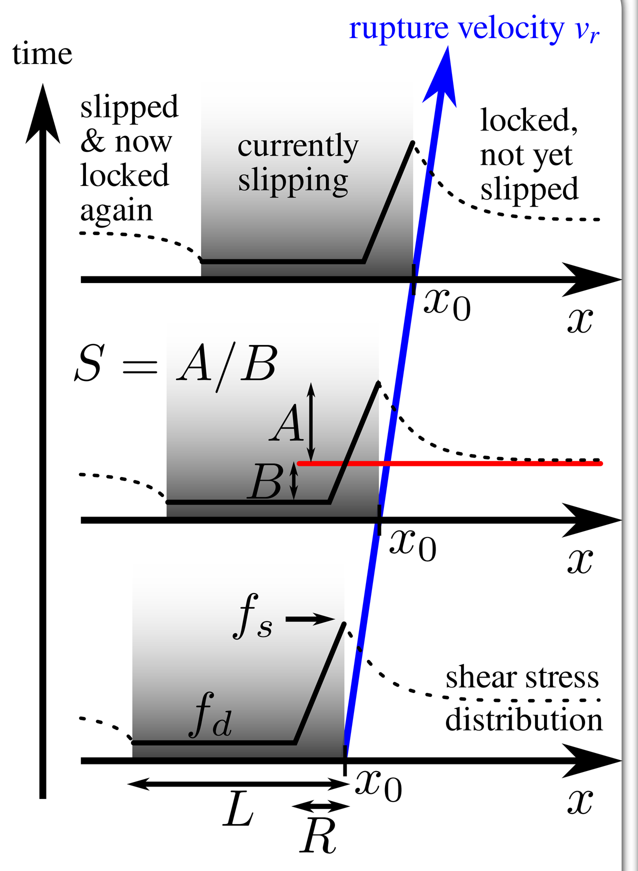

Fig. 2. Model setup of a pulse-like rupture suggested by Rice et al. [2005].

Parameterized by the following kinematical/mechanical quantities:

- L : length of the pulse
- R : length of the process zone
- x_0 : rupture front, i.e., the leading edge of the pulse
- v_r : rupture velocity, i.e., moving velocity of the pulse
- $\Delta\dot{u}$: slip velocity
- μ : the rigidity
- G : energy release rate due to propagation of the pulse
- f : friction
- $f_s := f(x_0)$: yield stress, i.e., the maximum static friction
- $f_d := f(x_0 - L)$: constant level of dynamic friction
- $f_0 := \lim_{x\to\infty} f$: background shear stress acting on the fault
- $S := \dfrac{f_s - f_0}{f_0 - f_d}$: stress ratio

Equation and solution

The system is moving with constant speed v_r and Galilean transformation can be applied, i.e., $x := x - v_r t$.

The transformation and superposition of elastic response to moving dislocation yields the following singular integral equation:

$$f(x) = f_0 + \frac{\mu F(v_r)}{v_r}\, p.\,v. \int_{x_0-L}^{x_0} \frac{\Delta\dot{u}(\xi)}{\xi - x}\frac{d\xi}{\pi}. \tag{1}$$

After solving eq.(1), Rice et al. [2005] obtained followings:

- a one-by-one relationship between S and $\frac{R}{L}$
- a trade-off among four parameters:

$$\Psi(S, G_c, R \text{ or } L, v_r) = 0 \tag{2}$$

Significant achievements explicitly and implicitly proposed by Rice et al. [2005]

- Dependence of Ψ on S is weak (a factor of two)
- G can be estimated using kinematic parameters L and v_r, which are determined by seismic inversion analysis. They concluded that $G \sim 0.1 - 4$ [MJ/m^2].
- **Process zone size R and rupture velocity v_r cannot be determined simultaneously even after the stress ratio S and fracture energy G_c are given due to the trade-off.**

2.2. Focusing on relaxation time of friction

Rice et al. [2005] assumed that the rupture velocity and length of the process zone is constant. As seen in Fig. 3., this means that

$$\tau := \frac{R}{v_r} \tag{3}$$

is also constant. Moreover, τ can be regarded as a prescribed parameter under a framework of Hatano [2009], which is not for linear elastic fracture mechanics but for statistical physics.

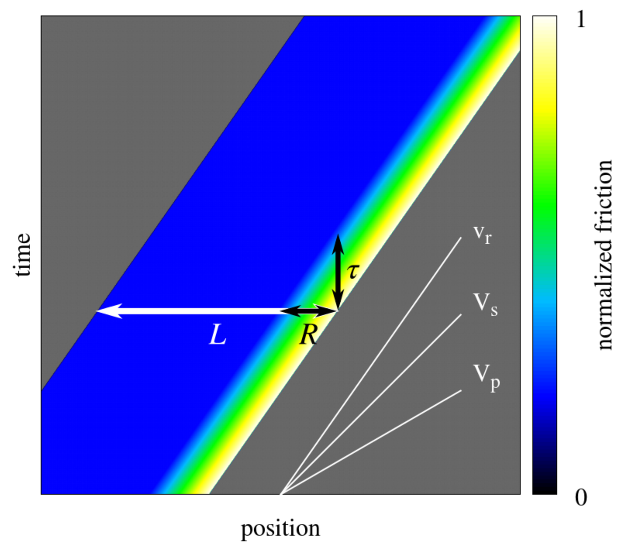

Fig. 3. Spatio-temporal distribution of friction on the pulse assumed in the model of Rice et al. [2005].

2.3. A time-weakening friction model by Hatano [2009]

Assumptions on the fault plane [Hatano 2009]

- Slipping plane corresponds to a granular layer sandwiched by two elastic bodies as shown in Fig. 4.
- Each elastic grain in the layer has the same size and has a collision with its surrounding grains and/or walls of the bodies.

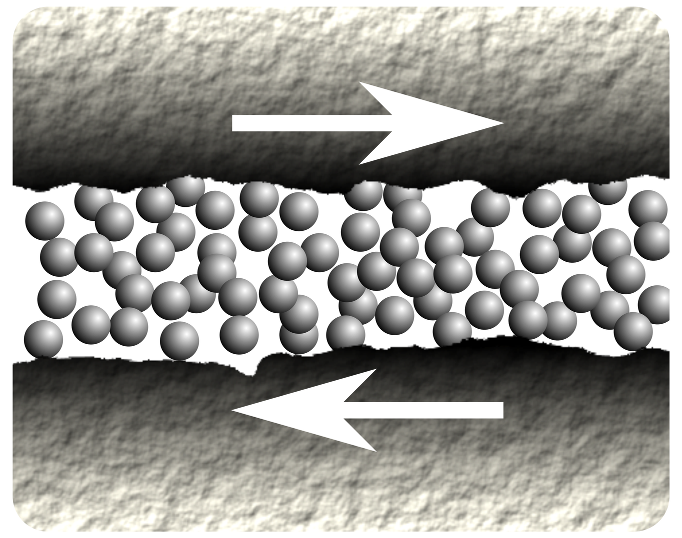

Fig. 4. Schematic illustration of a Granular layer modeled by Hatano [2009].

Parameterized by the following quantities:

- τ : relaxation time of friction, i.e., traction acting on the walls macroscopically
- c : numerical constant. ~ 1 if the grain size distribution is uniform, or unknown otherwise
- H_0 : thickness of the granular layer
- ρ : density
- P : pressure

Numerical solution of Hatano [2009]

As a result of Hatano's first principal calculation, τ is almost independent of externally enforced slip velocity and obeys the following:

$$\tau = cH_0\sqrt{\frac{\rho}{P}}. \tag{4}$$

2.4. Combining the pulse model of Rice et al. [2005] with the time-weakening friction model by Hatano [2009]

By substituting eq. (3) into eq. (2), v_r is determined uniquely for given value of τ. In other words, an intersection of a curve corresponding to eq. (2) and a straight line corresponding to eq. (3) shown in Fig. 5. indicates v_r and R.

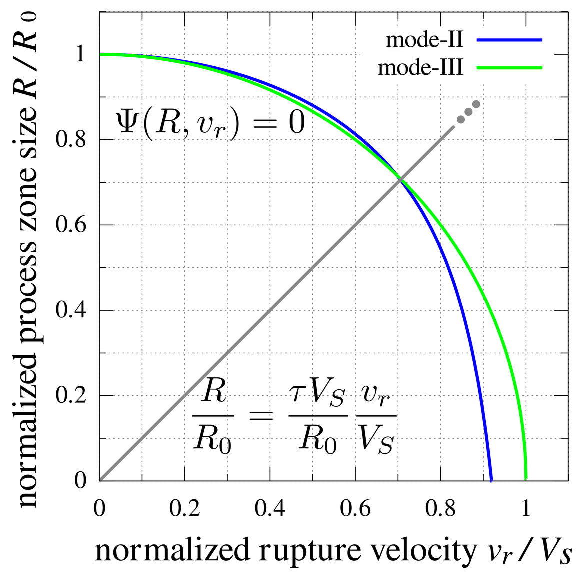

Fig. 5. Relations between v_r and R given in eqs. (2) and (4).

Moreover, by considering eq. (4), we obtain

$$\frac{\tau V_S}{R_0} = \frac{acH_0 P^{\frac{3}{2}}}{G}, \tag{5}$$

where the LHS is a slope of line in Fig. 5 and $a \sim 1 \times 10^{-6}$ depends on S, elastic constants, and difference between coefficients of the maximum static friction and dynamic friction.

Eq. (5) means that **rupture velocity depends on fracture energy, granular layer thickness, and pressure**

3. Results

3.1. Rupture velocity with given values of H_0 and P

Notes:

Each parameter has uncertainty, e.g., a factor of two, so very detail of the intersection should not be focused.

A meaningful value is order of $\tau V_S / R_0$, the slope of line.

$$\begin{cases} \tau V_S/R_0 \gg 1 & \Leftrightarrow v_r/V_L \sim 0, \\ \tau V_S/R_0 \sim 1 & \Leftrightarrow v_r/V_L \sim 0.7, \\ \tau V_S/R_0 \ll 1 & \Leftrightarrow v_r/V_L \sim 1, \end{cases} \tag{6}$$

where $V_L = V_S$ for mode-III and $V_L = V_{\mathrm{Rayleigh}}$ for mode-II.

Although Hatano [2009] suggested $c = 1$, grain size distribution is not uniform as he assumed but quite heterogeneous, e.g., power low. Hence we here consider both $c = 1$ and $c = 10$.

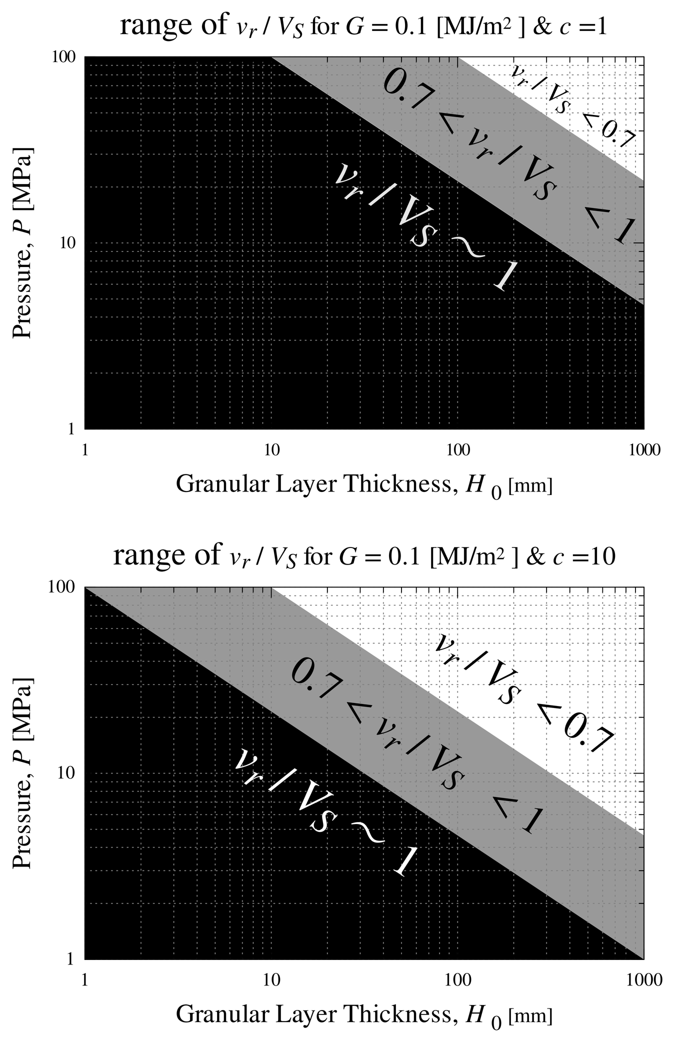

Fig. 6. Range of v_r/V_S for $G = 0.1$ [MJ/m^2] based on eq. (6). Boundaries indicate $\tau V_S/R_0 = 1$ for white/gray and $= 10$ for gray/black. The upper for $c = 1$ and the lower for $c = 10$.

4. Discussions & Conclusions

4.1. Assumption of $R/H_0 \gg 1$

Rice et al. [2005]

Thickness of the slipping plane is negligible.

Hatano et al. [2009]

The thickness is considered.

Conflict?

$R/H_0 \gg 1$ is required for combining them.

Valid for $c = 10$, but invalid for $c = 1$ and high pressure/low rupture velocity (Fig. 7).

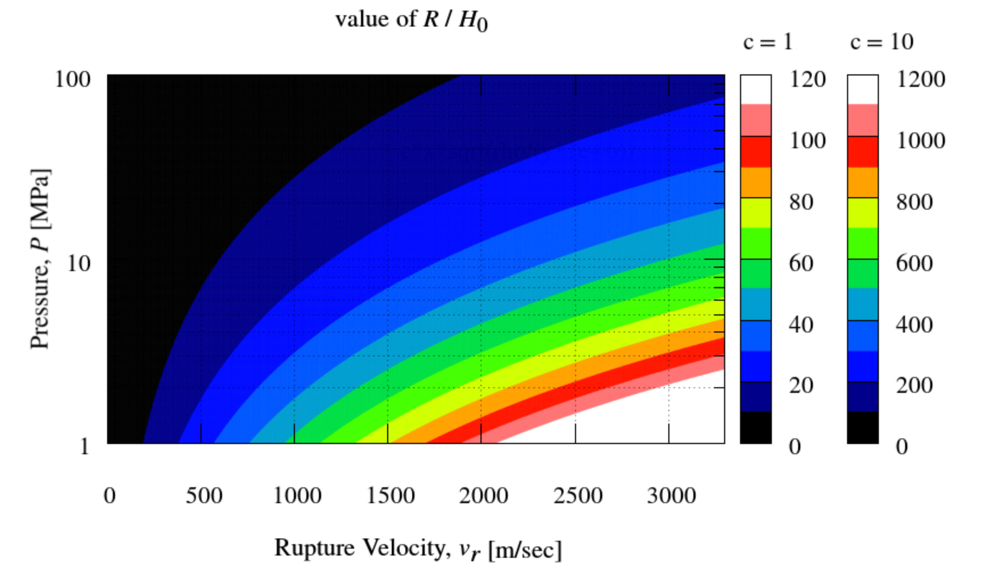

Fig. 7. Value of R/H_0. Left color bar is for $c = 1$ and Right one is for $c = 10$.

4.2. Determinability of a seismological unobservable

Table. 1. Determinability and observability of quantities.

Forward problem	Physical quantity	Seismic inversion
assumed	fracture energy G	observable
assumed	pressure in the crust P	unknown
assumed	granular layer thickness H_0	unknown
obtained	rupture velocity v_r	observable

If one of P and H_0 can be measured by some geological investigations, the other will be estimated.

Conclusions

Rupture velocity can be determined uniquely under the time-weakening friction of granular layer.

Rupture velocity depends on the granular layer thickness, pressure, and fracture energy.

Order of coefficient c for nonuniform grain size distribution is required.

Acknowledgement

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