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Dependence of Seismic Energy on Higher Wavenumber Components

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1. Introduction.

1.1. Motivations

$M_W + 2 \Rightarrow$ **"Energy"** $\times 1000$

Do they have a one-to-one relationship?

What's the "energy" here?

Estimation of the "energy"

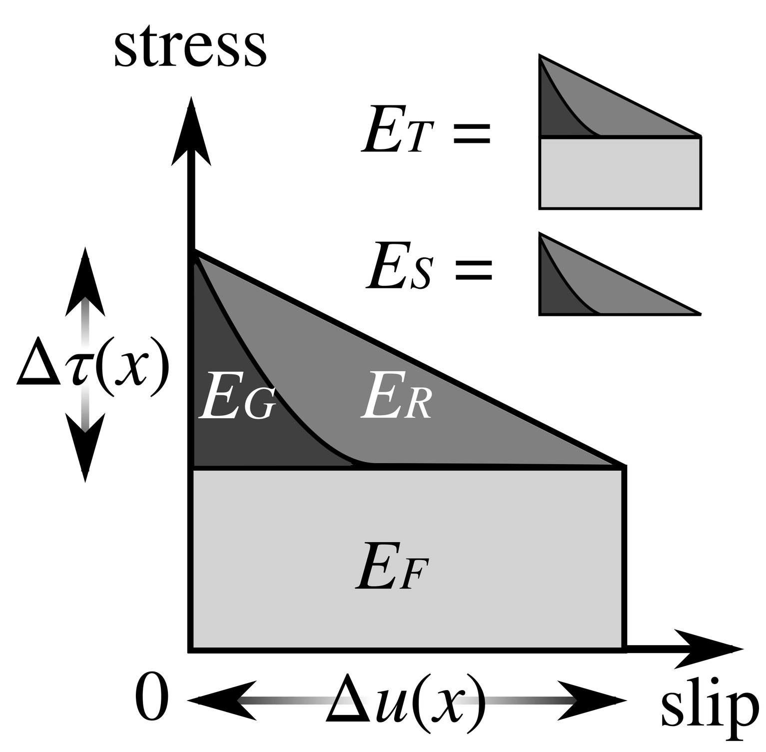
Traditional method works well under a certain assumption

Is this valid?

1.2. Background

Energy partitioning of the earthquake

For arbitrary point on the fault plane, density of strain energy release is given by slip amount, absolute value of stress, and static stress drop.



- x : Arbitrary point on the fault plane
- $\Delta u(x)$: Slip at x
- $\Delta\tau(x)$: Stress drop at x
- E_T : Total strain energy release from the entire region (inestimable due to E_F)
- E_F : Frictional heat (inestimable via seismic wave)
- E_R : Radiated energy
- E_G : Fracture energy
- $E_S = E_G + E_R$: Minimum of the strain energy release (Kanamori 1977)

$$E_T = \frac{1}{2}\int_{volume}\left\{\int_{initial}^{final}\tau d\epsilon\right\}dV$$
$$= \int_{fault}\Delta u\left(\tau^0 - \frac{1}{2}\Delta\tau\right)dS$$
$$= E_F + E_S,$$
$$\therefore E_S = -\frac{1}{2}\int \Delta u\,\Delta\tau\,dS,$$

where

- τ, τ_0 : absolute and initial stress
- $d\epsilon$: increment of strain

E_S and E_R should be estimated independently because E_G is significant and controversial.

1.3. Traditional concept

Kanamori 1977

The fault is assumed to be flat and embedded in an infinite homogeneous medium

$\Delta\tau(x)$ is assumed to be homogeneous along the fault plane

$$\Rightarrow E_S = -\frac{1}{2}\Delta\tau\int\Delta u\,dS$$
$$= -\frac{\Delta\tau}{2\mu}M_0$$

Robustness of estimation of M_0

because it is the low-wavenumber limit, i.e.,

$$\int_{fault}\Delta u\,dS = \lim_{\mathbf{k}\to 0}\int_{\mathbb{R}^2}\Delta u(\mathbf{x})\,e^{-i\mathbf{k}\cdot\mathbf{x}}\,d\mathbf{x}$$

1.4. Question

What about heterogeneous slip/stress drop?

2. Method

2.1. E_S in the wavenumber domain (Andrews 1980)

$\mathbf{k} = (k_1, k_2)$ **: wavenumber vector** ($k := |\mathbf{k}|$)

The fault is lying along x_1-x_2 plane

The fault is assumed to be embedded in an infinite homogeneous medium

slip direction is uniformly parallel to x_1-axis (i.e., $\Delta u_2 \equiv 0$)

$$\Rightarrow \mathfrak{F}\Delta\tau_{13}(\mathbf{k}) = \frac{1}{2}\frac{\mu}{k}\left(\frac{k_1^2}{1-\nu} + k_2^2\right)\mathfrak{F}\Delta u_1(\mathbf{k}),$$
$$\therefore E_S = -\frac{1}{2}\int_{\mathbb{R}^2}\Delta\tau(\mathbf{x})\,\Delta u(\mathbf{x})\,d\mathbf{x}$$
$$= -\frac{1}{2}\int_{\mathbb{R}^2}\overline{\mathfrak{F}\Delta\tau(\mathbf{k})}\,\mathfrak{F}\Delta u(\mathbf{k})\,d\mathbf{k}$$
$$= -\frac{1}{4}\int_{\mathbb{R}^2}\frac{\mu}{k}\left(\frac{k_1^2}{1-\nu} + k_2^2\right)|\mathfrak{F}\Delta u(\mathbf{k})|^2\,d\mathbf{k}$$

Especially, if $|\mathfrak{F}\Delta u(\mathbf{k})|^2$ depends only on $k := |\mathbf{k}|$

$$\Rightarrow E_S = \frac{\pi}{4}\frac{2-\nu}{1-\nu}\mu\int_0^\infty k^2|\mathfrak{F}\Delta u(k)|^2\,dk$$

E_S can be estimated if PSD of slip is given.

2.2. PSD of slip distribution (Mai & Beroza 2002)

von Karman-type PSD

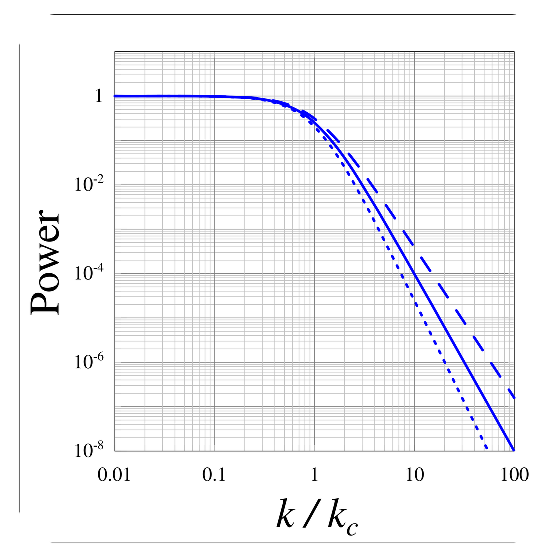
$$|\mathfrak{F}\Delta u(k)|^2 \sim \frac{M_0^2}{\mu^2\left\{1 + (k/k_c)^2\right\}^{1+H}}$$

is better for fitting many inversion results.

H : Hurst exponent

k-square model if $H = 1$.

42 of 44 inversion results provide that $H = 0.71 \pm 0.23$



2.3. Combining the above models

Inversion results are necessarily band-limited

The PSD for $k > k_{Max}$ cannot be obtained.

$$E_S(k < k_{Max}) = \frac{\pi}{4}\frac{2-\nu}{1-\nu}\mu\int_0^{k_{Max}}\frac{k^2M_0^2}{\left\{1 + (k/k_c)^2\right\}^{1+H}}dk$$
$$= \frac{\pi}{8}\frac{2-\nu}{1-\nu}\mu\,M_0^2\,k_c^3\,B\left(K;\frac{3}{2}, H - \frac{1}{2}\right),$$

where $K := \frac{1}{1+(k_{Max}/k_c)^{-2}}$, and $B(K; a, b) := \int_0^K t^{a-1}(1-t)^{b-1}dt$ is an incomplete beta function.

Dependency of estimated E_S on M_0, k_c, k_{Max}/k_c, and H can be investigated

2.4. Applicability to E_R

Estimation of radiated energy E_R

If PSD of radiated wave is modeled as $|A(f)|^2 \propto \frac{1}{\left\{1+(f/f_c)^2\right\}^{1+H}}$

$$\Rightarrow E_R(f < f_{Max}) \propto \int_0^{f_{Max}}\left|\dot{A}(f)\right|^2 df$$
$$= \int_0^{f_{Max}}\frac{f^2}{\left\{1 + (f/f_c)^2\right\}^{1+H}}df$$

ω-square model if $H = 1$.

"integration up to approximately ten times the corner frequency is necessary to approach 90% of the seismic energy" (Ide & Beroza 2001)

This should change if $H \neq 1$ (e.g., $H \sim 0.7$ for deeper region of the 2011 Tohoku-oki earthquake, Yagi et al. 2012)

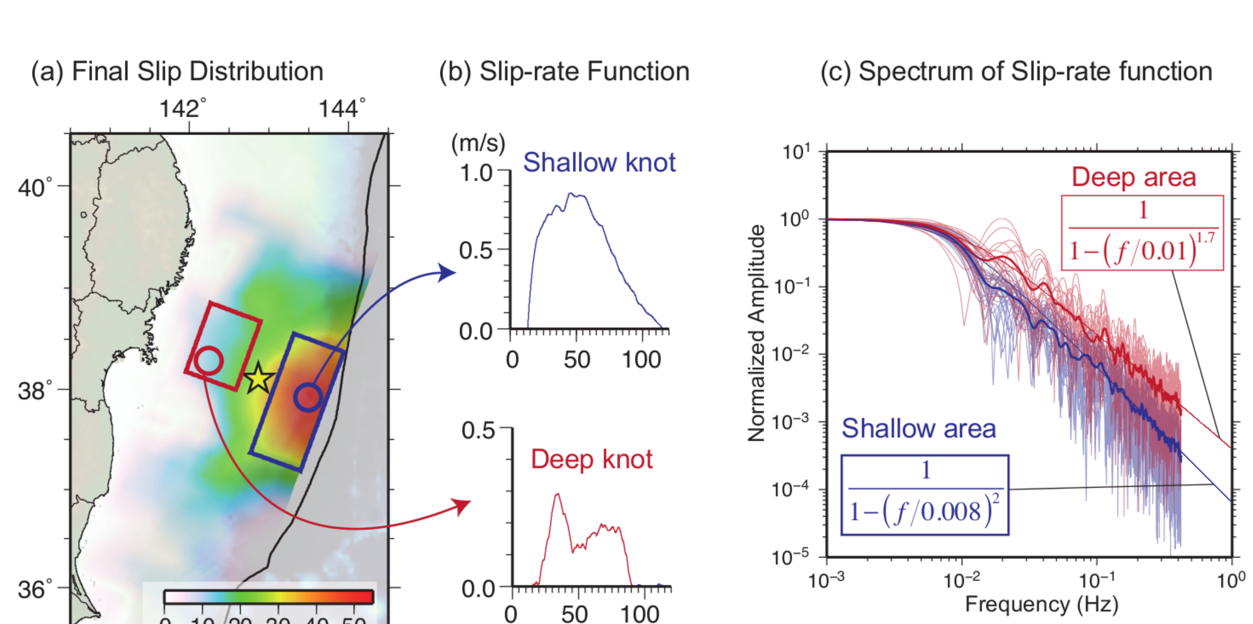
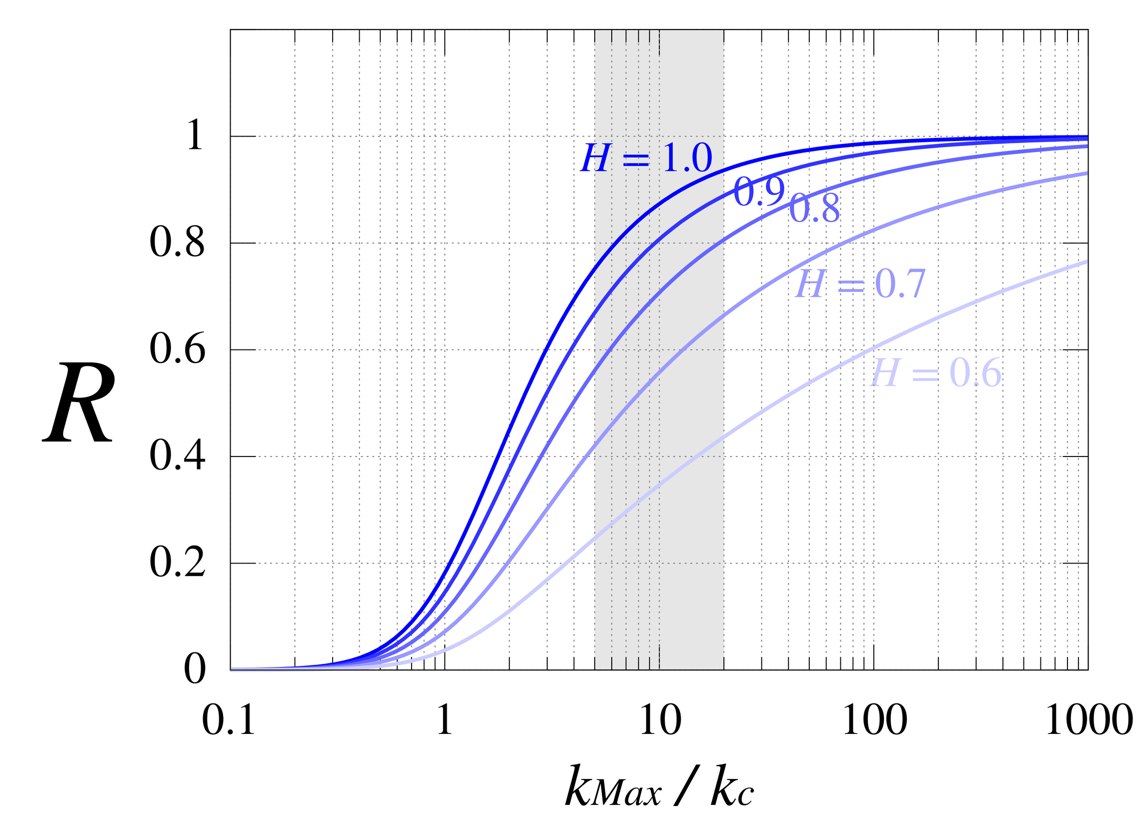


Figure 8 of Yagi et al. 2012 EPSL

3. Results

3.1. Dependence on H and k_{Max}/k_c

$$R := \frac{E'(k < k_{Max})}{E'(k < \infty)},$$ where E' indicates E_S or E_R.



$k_{Max}/k_c \sim 10$ **allows...**

$\sim 90\%$ of E_S or E_R if $H = 1$ (Ide & Beroza 2001).

$\sim 55\%$ of E_S or E_R if $H = 0.7$.

$k_{Max}/k_c \sim 400$ **allows...**

$\sim 100\%$ of E_S or E_R if $H = 1$.

$\sim 90\%$ of E_S or E_R if $H = 0.7$.

4. Discussions & Conclusions

4.1. Estimation accuracy of magnitude

Define magnitude as $M := \frac{2}{3}\log_{10}E' + C$**,**

where E' indicates E_S or E_R.

$$M = \frac{2}{3}\log_{10}E_S + C = \frac{2}{3}\log_{10}M_0 + \frac{2}{3}\log_{10}\frac{\Delta\tau}{2\mu} + C$$

is the moment magnitude if $\Delta\tau(x)$ is homogeneous.

$$M = \frac{2}{3}\log_{10}E_R + C$$

is the energy magnitude (Choy et al. 2006).

Difference in the magnitude for the von Karman-type PSD

$M(k < \infty) - M(k < 10k_c) \sim 0.3$ if $H = 0.7$.

$M(H = 1) - M(H = 0.7) \sim -0.15$ if $k_{Max} = \infty$.

4.2. Importance of higher wavenumber components

Present

E_S is determined using the low-wavenumber limit...

Invalid because stress drop is quite heterogeneous!

The best

Take sufficiently higher wavenumber (or frequency) components into account

The next-best

Do grid search and find k_c and H after seismic slip inversion analysis

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